

# Admissibility Without Projection: Empirical Identification of a Robust Non-Projecting Shell in the UNNS Substrate

## Abstract

We report the results of Chamber XIV-B, an experimental study within the Unbounded Nested Number Sequences (UNNS) program designed to isolate admissibility independently of projection. By systematically varying scale, resolution, noise, operator strength, and grid refinement, we identify a narrow admissible regime that is robust under perturbation and refinement, yet never projects to a stable structure. Extending this parametric analysis to operator composition in Chamber XXXVII, we establish a four-class taxonomy of operator admissibility, revealing that ensemble behavior is the norm and that admissibility can exist without realizability. This establishes an empirical separation between the existence of admissible structure and its realizability as a projection, motivating operator-level investigations beyond parametric control.

## 1 Introduction

In many physical and mathematical frameworks, the existence of an admissible configuration is tacitly assumed to imply the possibility of a realizable or stable representation. Within the UNNS program, this assumption is explicitly questioned. UNNS distinguishes between *admissibility*, understood as internal consistency under a given operator grammar, and *projection*, understood as the emergence of a stable, externally representable structure.

Chamber XIV-B was constructed to empirically test whether admissibility can persist in the absence of projection, and whether failure to project can be attributed to tunable external parameters or instead reflects a deeper structural obstruction.

## 2 Experimental Framework

Chamber XIV-B evaluates configurations using two invariant quantities:

- $\Delta_{\text{scale}}$ , measuring deviation from scale-consistent structure,
- $\Pi$ , a coherence or closure invariant.

Each configuration is classified as:

- *stable*,
- *partial* (admissible but non-projecting),
- or *unstable*,

according to fixed thresholds applied uniformly across all runs.

No additional observables or post-hoc criteria are introduced during validation.

### 3 Explored Axes

The chamber systematically varies five independent axes:

1. Scale control  $\mu$ ,
2. Resolution depth  $\Omega$ ,
3. Noise amplitude  $\sigma$ ,
4. Operator strength  $\lambda$ ,
5. Grid resolution.

Each axis is explored independently while holding the remaining parameters fixed, allowing attribution of observed effects to specific controls.

## 4 Results

### 4.1 -Localized Admissibility

Admissibility is observed only within a narrow interval of  $\mu$ . Outside this band,  $\Delta_{\text{scale}}$  exceeds the admissibility threshold and configurations become unstable. Throughout this transition,  $\Pi$  remains high, indicating that loss of admissibility is driven by scale inconsistency rather than coherence collapse.

This establishes  $\mu$  as a gate for admissibility.

### 4.2 Resolution Saturation

Increasing resolution depth  $\Omega$  beyond a moderate threshold produces no significant change in  $\Delta_{\text{scale}}$  or  $\Pi$ . The admissible regime is therefore resolution-insensitive, indicating that failure to project is not due to insufficient refinement.

### 4.3 Grid Robustness

Grid refinement over multiple resolutions preserves classification and invariant structure. The observed admissible regime is not a discretization artifact.

### 4.4 Noise Robustness

Noise perturbations up to  $\sigma \approx 0.05$  deform invariants smoothly without inducing instability or projection. Only at substantially larger noise levels does admissibility fail, defining a clear refusal boundary. Noise therefore does not control projection within the admissible regime.

### 4.5 Tangential Role of Operator Strength

Variation of operator strength  $\lambda$  within tested bounds deforms invariants continuously and improves coherence, but does not induce projection or instability.  $\lambda$  is tangential within the admissible regime.

## 5 Key Finding

Across all explored axes, Chamber XIV-B identifies a regime that is:

- admissible under invariant criteria,
- localized in  $\mu$ ,
- invariant under resolution refinement,
- robust under noise and operator-strength variation,
- persistently non-projecting.

This constitutes a *robust admissible shell* that does not give rise to stable projection.

## 6 Interpretation

The results demonstrate an empirical asymmetry between existence and characterization. Admissibility alone does not guarantee realizability. Because projection failure persists under exhaustive parametric variation, it cannot be attributed to tuning choices or insufficient resolution.

The obstruction to projection must therefore arise at the level of operator composition or structural grammar, rather than from external parameters.

## 7 Scope and Limitations

Chamber XIV-B does not identify the operator-level mechanism responsible for non-projection. It does not propose new physical laws, nor does it challenge the empirical success of existing theories. Its role is diagnostic: to isolate admissibility as a measurable and distinct concept within the UNNS substrate.

## 8 Chamber XXXVII: Operator Composability Results

### 8.1 Overview

Chamber XXXVII investigates whether admissibility identified at the parametric level persists under operator composition. Building on the admissible shell established in Chamber XIV-B, this chamber tests composability as a property of operator grammar rather than external parameter tuning.

The focus is not on discovering new stable structures, but on determining whether admissible configurations can be realized under composition in a reproducible and invariant-preserving manner.

### 8.2 Experimental Outcome

Across all tested configurations spanning approximately 410 realizations, the following results are observed:

- The base operator  $\tau$  satisfies its admissibility gate for all realizations and all tested depths (360/360 seeds,  $G_\tau = 100\%$ ).

- The  $\sigma$  operator exhibits high ensemble admissibility: structure is preserved in approximately 75% of realizations ( $G_2 = 0.75$ ,  $N=180$ ), with compositional benefit observed in 55% ( $G_o = 0.55$ ).
- The  $\kappa$  and  $\phi$  operators exhibit low ensemble admissibility: structure is preserved in approximately 30% of realizations ( $G_2 = 0.30$ ,  $N=105$  each), but compositional benefit is *never* observed ( $G_o = 0.00$  across all seeds).

Importantly, no operator exhibits complete inadmissibility ( $G_2 = 0$ ) within the tested regime, suggesting that hard structural refusal may be rare or absent in the admissible shell identified by Chamber XIV-B.

### 8.3 Seed Dependence

For all ensemble-admissible operators ( $\sigma, \kappa, \phi$ ), the identity of realizations that satisfy the composability gates varies across seeds. However, the aggregate pass rates remain invariant under increased iteration depth and independent replication.

This establishes a sharp distinction between realization-level variability and ensemble-level stability. No deterministic criterion based on invariant magnitude, resolution, or operator strength predicts composability on a per-seed basis.

### 8.4 Revised Operator Taxonomy

The combined results of Chambers XIV-B and XXXVII reveal that admissibility and realizability are independent dimensions.

Empirically, four distinct operator classes are observed:

1. **Deterministic admissibility** ( $G_2 = 100\%$ ): All realizations preserve structure. *Example:  $\tau$ .*
2. **Ensemble-realizable admissibility** ( $0 < G_2 < 100\%$ ,  $G_o > 0$ ): Structure is preserved in measure and composition provides benefit for a stable subset of realizations. *Example:  $\sigma$  with  $G_2 = 75\%$ ,  $G_o = 55\%$ .*
3. **Ensemble-futile admissibility** ( $0 < G_2 < 100\%$ ,  $G_o = 0$ ): Structure is preserved in measure, but composition never improves upon the base operator. *Examples:  $\kappa$  and  $\phi$ , both with  $G_2 = 30\%$ ,  $G_o = 0\%$ .*
4. **Inadmissibility** ( $G_2 = 0\%$ ): Structure is never preserved. *No instances observed in this study.*

These classes are empirically distinguishable and invariant under stress.

### 8.5 The Admissibility–Realizability Gap

Chamber XXXVII demonstrates that admissibility does not imply realizability. Operators  $\kappa$  and  $\phi$  preserve structure in a nonzero fraction of realizations ( $G_2 \approx 30\%$ ), yet never produce beneficial composition ( $G_o = 0\%$  across 105 seeds each).

This establishes a fundamental separation between:

- *can compose* (admissibility), and

- *should compose* (realizability).

The existence of ensemble-futile operators shows that admissibility alone is insufficient as a criterion for structural usefulness. This phenomenon is invariant under stress, depth, and seed replication, and therefore reflects a property of operator grammar rather than parameter choice.

Remarkably,  $\kappa$  (curvature equalization) and  $\phi$  (topological folding) exhibit identical statistics: both show  $G_2 = 29.5\%$  and  $G_o = 0.0\%$ . This suggests they share a common structural limitation within the  $\tau$ -stabilized regime, possibly reflecting a universal boundary for variance-reduction operators.

## 8.6 Summary of Composability Results

Table 1 summarizes all validated results across approximately 410 realizations.

Operator	$G_\tau$	$G_2$	$G_o$	N	Classification
$\tau$	100%	—	—	360	Deterministic
$\sigma$	100%	75%	55%	180	Ensemble-realizable
$\kappa$	100%	30%	0%	105	Ensemble-futile
$\phi$	100%	30%	0%	105	Ensemble-futile

Table 1: Validated operator composability results.  $G_\tau$  measures  $\tau$ -admissibility,  $G_2$  measures secondary operator admissibility,  $G_o$  measures compositional benefit. All pass rates stable under stress testing ( $M=225$  vs  $M=300$ :  $\Delta = 0.000$  for  $\sigma$ ). 95% confidence intervals:  $\pm 3\%$  for  $\kappa, \phi$ ;  $\pm 5\%$  for  $\sigma$ .

## 8.7 Interpretation

The results demonstrate that admissibility and realizability are not equivalent. An operator may preserve admissibility in measure while failing to do so deterministically ( $\sigma$ ), or preserve admissibility without ever producing benefit ( $\kappa, \phi$ ).

For  $\sigma$ , composability depends on whether a minimal secondary structure survives composition. Whether this occurs is realization-dependent, yet the measure of successful realizations (75%) is robust under stress.

For  $\kappa$  and  $\phi$ , admissibility depends on structure preservation (30% success rate), but *realizability* depends on whether that preservation adds value (0% success rate). The gap between these rates reveals that structural validity and operational utility are independently constrained.

This behavior cannot be attributed to parameter misconfiguration, numerical instability, or insufficient refinement. It reflects intrinsic properties of the operator grammar itself.

Notably, the absence of inadmissible operators ( $G_2 = 0$ ) in our study suggests that within the admissible shell established by Chamber XIV-B, all operators can compose successfully in *some* measure. The variation lies not in whether composition is possible, but in how often it succeeds and whether it provides value.

## 8.8 Relation to Chamber XIV-B

Chamber XIV-B established that admissibility persists under exhaustive parametric variation without inducing projection. Chamber XXXVII complements this result by showing that even within the admissible shell, realizability under composition is operator-dependent.

Together, the chambers demonstrate that:

- admissibility is not sufficient for realizability,
- parametric exhaustion does not guarantee composability,
- operator grammar introduces additional, irreducible constraints.

## 9 From Parametric Admissibility to Ensemble Realizability

### 9.1 Motivation

Chamber XIV-B established that admissibility can persist under exhaustive parametric variation without inducing projection. Within a narrow  $\mu$ -localized shell, invariant criteria are satisfied robustly under changes in resolution, noise, operator strength, and discretization. However, no stable projection emerges anywhere within this shell.

This raises a sharper question: *Does admissibility guarantee realizability under operator composition?*

Chamber XXXVII was constructed to address this question by testing admissibility at the level of operator grammar rather than external parameters.

### 9.2 Admissibility Classes in UNNS

The combined results of Chambers XIV-B and XXXVII motivate a refinement of admissibility within the UNNS framework.

**Definition (Ensemble Admissibility).** Let  $\mathcal{O}$  be an operator acting on a UNNS substrate under fixed invariant thresholds and admissibility criteria. For a fixed configuration class  $C$ , define the seed space  $\mathcal{S}$  as the set of admissible realizations.

The operator  $\mathcal{O}$  is said to be *ensemble-admissible* with respect to a gate  $G$  if:

1.  $G$  is satisfied on a nonzero-measure subset of  $\mathcal{S}$ ,
2. the measure of this subset is stable under refinement and stress,
3. no deterministic criterion guarantees  $G$  pointwise over  $\mathcal{S}$ .

Ensemble admissibility is strictly weaker than deterministic admissibility and strictly stronger than inadmissibility.

### 9.3 UNNS Proposition XXXVII-A: Ensemble Admissibility of $\sigma$

**Proposition (Ensemble Admissibility of  $\sigma$ ).** Within the admissible shell identified in Chamber XIV-B, the  $\sigma$  operator admits composition with secondary structure  $\Omega_2$  in an ensemble-admissible manner.

Specifically:

1. The base operator  $\tau$  satisfies admissibility gates for all tested realizations.
2. Composition with  $\sigma$  preserves  $\tau$  deterministically ( $G_\tau = 100\%$  in all contexts).
3. Composition with  $\sigma$  preserves  $\Omega_2$  and satisfies composability gates on a stable, nonzero-measure subset of realizations ( $G_2 = 75\%$ ,  $G_\circ = 55\%$ ).

4. Realization-level variability persists under increased depth and independent replication, yet aggregate pass rates remain invariant ( $\Delta = 0.000$  under  $M=225 \rightarrow 300$  stress).

Therefore,  $\sigma$  is neither deterministically composable nor structurally forbidden, but occupies a distinct admissibility class characterized by ensemble-level realizability.

**Corollary (Ensemble-Futile Operators).** Within the same admissible shell, the  $\kappa$  (curvature equalization) and  $\phi$  (topological folding) operators exhibit ensemble admissibility at approximately 30%, but zero realizability ( $G_o = 0.00$  across 105 seeds each). This establishes the existence of a distinct *ensemble-futile* class: operators that preserve structure in measure without providing compositional benefit.

The identical pass rates for  $\kappa$  and  $\phi$  ( $G_2 = 29.5\%$  for both) suggest a universal constraint for variance-reduction operators in  $\tau$ -stabilized regimes, motivating future investigation of the mechanistic basis for this boundary.

## 9.4 Interpretation

Taken together, Chambers XIV-B and XXXVII establish multiple distinct concepts within the UNNS framework:

1. *Parametric admissibility*: Established by Chamber XIV-B within a narrow  $\mu$ -localized shell, robust under resolution and noise perturbation.
2. *Operator-level admissibility*: Ranges from deterministic (100% for  $\tau$ ) through high ensemble (75% for  $\sigma$ ) to low ensemble (30% for  $\kappa, \phi$ ). May exist only in measure, not pointwise.
3. *Compositional realizability*: Independent constraint beyond admissibility. Even when structure is preserved ( $G_2 > 0$ ), composition may provide no benefit ( $G_o = 0$  for  $\kappa, \phi$ ).

The absence of inadmissible operators ( $G_2 = 0$ ) suggests that within the parametric admissible shell, hard structural refusal is rare or absent. Instead, we observe a spectrum from high-value composition ( $\sigma$ ) through futile composition ( $\kappa, \phi$ ) to no composition ( $\tau$  alone).

## 9.5 Implications

These results demonstrate that admissibility and realizability are fundamentally distinct and independently constrained concepts within UNNS. An admissible structure may exist robustly without being deterministically realizable under composition ( $\sigma$ ), or may be realizable without providing operational benefit ( $\kappa, \phi$ ).

The four-class taxonomy provides an empirical characterization of operator compatibility:

- Deterministic operators ( $\tau$ ): universal admissibility
- Ensemble-realizable operators ( $\sigma$ ): valuable in measure
- Ensemble-futile operators ( $\kappa, \phi$ ): admissible but valueless
- Inadmissible operators: potentially absent in admissible regimes

This framework motivates investigation of: (1) what determines ensemble fraction (30% vs 75%), (2) what separates realizability from admissibility, (3) whether the 30% boundary reflects a universal constraint, and (4) how operator properties predict compositional class.

## 9.6 Conclusion

Chamber XXXVII establishes ensemble admissibility as a distinct and measurable mode within the UNNS framework, with the discovery of ensemble-futile operators revealing a fundamental gap between structural admissibility and operational realizability.

The  $\sigma$  operator occupies a boundary class between deterministic admissibility and structural refusal, while  $\kappa$  and  $\phi$  demonstrate that structure preservation does not guarantee utility.

These results motivate further investigation of operator grammar while standing independently as validated empirical findings.

Chamber XIV-B provides direct empirical evidence that admissible structure can exist without realizable projection. By exhaustively eliminating parametric explanations, it establishes the necessity of operator-level analysis for understanding realizability within UNNS.

This result motivates subsequent chambers focused on composability and operator grammar, while standing independently as a validated experimental finding.

## A Methods: Chamber XIV-B Protocol

Chamber XIV-B implements a fixed evaluation kernel that maps external parameters to invariant diagnostics. No adaptive tuning, learning, or post-selection is employed during evaluation.

### A.1 Invariant Computation

For each configuration, the chamber computes:

- $\Delta_{\text{scale}}$ , a scalar measure of deviation from scale-consistent structure across nested resolution levels,
- $\Pi$ , a coherence invariant measuring closure consistency under internal operator application.

These invariants are computed deterministically for a given parameter set and random seed.

### A.2 Classification Rule

Each configuration is classified using fixed thresholds:

- **Stable**:  $\Delta_{\text{scale}} < 10^{-3}$ ,
- **Partial**:  $10^{-3} \leq \Delta_{\text{scale}} < 6.5$  and  $\Pi \geq 0.6$ ,
- **Unstable**: otherwise.

The thresholds are held constant across all validation runs. No reclassification or adaptive thresholding is performed.

### A.3 Exploration Axes

The chamber explores admissibility by varying one axis at a time:

- $\mu$ : scale control parameter,
- $\Omega$ : resolution depth,

- $\sigma$ : noise amplitude,
- $\lambda$ : operator strength,
- grid resolution.

All remaining parameters are held fixed during each sweep.

#### A.4 Randomization and Reproducibility

Each configuration is evaluated across multiple random seeds. Classification outcomes reported in this work are invariant across seeds within the admissible regime. No seed-dependent bifurcations were observed.

#### A.5 Stopping Criteria

Exploration along a given axis is terminated when either:

- invariants saturate under further refinement, or
- admissibility fails (transition to unstable classification).

No attempts are made to recover stability once admissibility is lost.

### B Methods: Chamber XXXVII Protocol

Chamber XXXVII implements composability testing through systematic seed-batch evaluation.

#### B.1 Gate Definitions

Three composability gates are evaluated for each realization:

- $G_\tau$ : Base operator  $\tau$  admissibility ( $CR_\tau < 1$  and  $\Delta_\tau \leq \delta_\tau$ )
- $G_2$ : Secondary operator admissibility ( $CR_2 < 1$  and  $\Delta_2 \leq \delta_2$ )
- $G_o$ : Compositional benefit ( $CR_o < CR_\tau$ , strict improvement)

Thresholds  $\delta_\tau = \delta_2 = 0.15$  are held constant across all experiments.

#### B.2 Batch Evaluation

For each operator, N seeds are evaluated independently:

- $\tau$ : N=360 (universal baseline)
- $\sigma$ : N=180 at M=225 and M=300
- $\kappa$ : N=105 at M=225
- $\phi$ : N=105 at M=225

Pass rates are computed as the fraction of seeds satisfying each gate.

### B.3 Stress Testing

For  $\sigma$ , stress invariance is verified by comparing  $M=225$  vs  $M=300$  (33% increase in iteration depth) across identical seed ranges. Perfect invariance ( $\Delta = 0.000$  for all gates) confirms ensemble admissibility as a structural property.

## C Validated Data Summary

This appendix summarizes the datasets used to validate both chambers. All data were generated using frozen chamber versions (`chamber_xiv_b_v1.0.11.html`, `chamber_xxxvii_v0.3.0.html`).

### C.1 Chamber XIV-B: Parameter Sweeps

The following sweeps were performed and validated:

- **$\mu$ -sweep:** Identification of a narrow admissible band centered near  $\mu \approx 1.65$ . Outside this band,  $\Delta_{\text{scale}}$  exceeds the admissibility threshold despite high  $\Pi$ .
- **$\Omega$ -sweep:** Resolution depth varied from  $\Omega \approx 200$  to  $\Omega \approx 800$ . Invariant values saturate rapidly and remain stable under further refinement.
- **$\sigma$ -sweep:** Noise amplitude varied from  $\sigma = 0$  to  $\sigma = 0.05$ . Admissibility persists with smooth invariant deformation. A refusal boundary is observed only at substantially larger  $\sigma$ .
- **$\lambda$ -sweep:** Operator strength varied over  $\lambda \in [0.05, 0.20]$ . Invariant deformation is smooth and tangential; no projection or instability is induced.
- **Grid refinement:** Grid resolution varied over multiple scales. Classification and invariant structure are preserved, confirming resolution independence.

### C.2 Chamber XXXVII: Composability Data

Each operator batch produces structured JSON exports containing:

- parameter configuration,
- gate outcomes ( $G_\tau, G_2, G_\circ$ ),
- batch summary (pass rates),
- per-seed results with invariants.

The complete dataset consists of:

- $\sigma$ : 6 batches ( $M=225,300$ ;  $N=20,55,105$ ) totaling 360 seeds
- $\kappa$ : 1 batch ( $M=225$ ;  $N=105$ )
- $\phi$ : 1 batch ( $M=225$ ;  $N=105$ )

All files are archived alongside the chamber implementations and are sufficient to reproduce every conclusion reported in this paper.

### C.3 Validation Criteria

A result is considered validated if:

- gate outcomes are stable across independent batches,
- pass rates vary smoothly or remain invariant under parameter changes,
- no hidden parameter coupling is detected,
- results persist under stress testing where applicable.

All reported findings satisfy these criteria.